EXPERIMENTAL VERIFICATION OF THE CRITERIA OF NONUNIFORMITY IN A FLUIDISED BED

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It is shown that a suitable measure of the degree of nonuniformity in fluidised beds is the simplex $\Delta L/L_0$. The values of this simplex were evaluated from measurements in beds fluidised by water or air. A quantitative comparison of the quantity $\Delta L/L_0$ with the criteria of other authors has shown that in water — fluidised systems the closest relation exists between $\Delta L/L_0$ and the criterion φ, respect. the Froude number Fr. In systems fluidised by air, the closest relation exists between $\Delta L/L_0$ and the criteria k, $\Delta x/M$, φ and Fr. The maximum value of the sample correlation coefficient, 0·78 and 0·86 for systems fluidised by water and air respectively, indicates that no criterion compared can match $\Delta L/L_0$ as a measure of the degree of nonuniformity of the fluidised bed.

The difficulties of a complex analysis of the hydrodynamics of a nonuniformly fluidised bed have stimulated an effort to express the degree of nonuniformity of these beds by means of a variety of criteria or characteristics of nonuniformity and to determine their dependences on selected measurable parameters characterizing the bed. This effort is further stimulated by the need to work out a quantitative test of the quality of fluidisation in industrial equipment, to form the system of automatic control and to achieve a sufficient accuracy of the scale-up from a laboratory experiment. As a suitable measure of the degree of nonuniformity in small diameter columns may be taken the simplex $\Delta L/L_0$ defined by the following equation

$$\Delta L/L_0 = (L_{max} - L_{min})/L_0 \,. \tag{1}$$

The experimentally determined values of this simplex are compared qualitively and quantitatively with the following criteria: Fr (Wilhelm, Kwauk[1]), φ (Wicke, Hedden[2]), k (Tjurjajev, Cajlingold, Bujlov[3]), d_B/d (Harrison, Davidson, de Kock[4]), $\Delta x/M$ (Jackson[5]), m (Rietema[6]), S (Molerus[7]).

THEORETICAL

The balance of forces acting on individual particles or larger volume elements of real fluidised beds is constantly being upset as a consequence of origination and decay of the disturbing forces. The disturbation forces may act periodically or singly on either all particles of the bed or only locally. The primary local disturbance can trigger a series of the secondary disturbances or decay. The disturbances may be given rise to by irregularities in the flow of fluid near the grid and the walls of the equipment, by pulsations of the fluid supplied into the equipment, by mechanical interferences, by deformation of the velocity profile at propagation the primary disturbances, by the turbulency of the flow, *etc.*

The question of the origin, the magnitude and the effect of the disturbances on the equilibrium of the fluidised beds has not been solved to date. To make a correct approach to the problem, it should be ascertained whether a uniformly fluidised bed is *a priori* in a stable or unstable equilibrium.

From experimental observation in a water–solids system it is apparent that the fluidised beds are in stable equilibrium. That is, one can realize a fluidised bed in which the particles do not move except for oscillations about an equilibrium position. That means that only small disturbances act on particles causing thus their oscillations without perturbing the structure of the bed, which is in accord with the definition of the stable equilibrium. However, every equilibrium can be disturbed by superimposing a sufficiently large disturbance. From experimental observation it follows that the disturbing forces are greater in systems fluidised by gases than those fluidised by liquids. The occurence, location and intensity of the disturbance possesses a random character.

As a consequence of the action of sufficiently large disturbances of any kind the space distribution of particles corresponding to the equilibrium distribution is being disturbed. This gives rise to the formation of arrangements which we shall term free configurations and aggregates. Under the term free configurations we mean local increase in concentration of particles, preserving the contact of individual particles of the configuration with the flowing fluid. The beds of this type are common in systems fluidised by liquids.

In the aggregate the particles touch each other. During its life the aggregate behaves as a particle which can be assigned characteristic parameters. The fluid mostly by-passes the aggregate and only to a minor extent penetrates through the interior of the aggregate. Formation of the aggregates is typical for systems fluidised by gases. There are always several sources of disturbances and their mathematical expression has not been found thus far. Our considerations regarding the effect of the disturbances are based on the experimental and theoretical study of the problem.

At incipient fluidisation the particles are packed in the bed so closely that a slow gradual decrease of velocity of the fluid without shaking results in a stagnant bed of the minimum-fluidisation voidage (for spherical particles ε_p 0·42). Thus the transition of a uniformly fluidised bed into a nonuniform one is very unlikely. The expansion equations for uniformly fluidised beds hold generally in region of incipient fluidisation with the exception of the chanelling beds. In the following we shall concentrate only on beds above the minimum-fluidisation voidage, *i.e.* for $w > w_p$.

The primary disturbances in real systems have always a local character, *i.e.* they cause changes of concentration always only in their immediate neighborhood. The mechanism of propagation of a primary disturbance can be demonstrated by the model of Todes[8]. According to that model a pressure gradient forms whenever a certain nonuniformity occurs and the particles begin to approach each other by the action of pressure forces. The new velocity field in the neighborhood of the particle causes the change of the resistance and upsets the balance between the resistance forces and the effective weight of the particle. Ultimately, the free configurations or aggregates form which either rise or fall in the volume of the bed.

The perturbation of the particle distribution in a certain part of the bed causes other disturbances in the vicinity to appear, which either spread into the whole volume or decay. In the proximity of the level the falling particles form the deepenings and the rising particles the peaks. The height of the peaks and the deepenings in a certain instant reflects not only the character of the layer near the level but also the changes that took place in preceding time instant in a certain depth. The total height characterizes the integral effect of the disturbance forces acting in the bed at a given moment. The integral effect may be characterized by the quantity $\Delta L' = L'_s - L_r$, expressing the deviation of the mean height, L'_s, from the height L_r of the corresponding uniformly fluidised bed (Fig. 1).

The fluctuations of the pressure loss, the averaged values of voidage, density and the height of the bed $L'_s(\tau)$ possess a random character in time τ, and, on the basis of investigation of several authors[3,8,9], at $w =$ const. may be regarded as stationary random processes. If only free configurations form at $w =$ const., one can expect in sufficiently long period of time such distribution of probability of fluctuations $\Delta L'$ that the probability mean value $\overline{L'_s} = L_r$, and $\overline{\Delta L'} = 0$. This condition is fulfilled *e.g.* by the following model. The forces causing contraction of the volume of the bed are followed by forces causing corresponding expansion of the volume of the bed. Both these forces possess a hydrodynamic character. It is apparent that $\overline{\Delta L'}$ cannot be a measure of nonuniformity.

In beds in which the aggregates and bubbles of fluid form ($w =$ const.), the time averaged quantities characterizing the bed are not identical in general with those characterizing the uniformly fluidised because a part of the fluid passes through the bed as bubbles. Generally, in beds with aggregates $\overline{\Delta L'} \neq 0$ and the relation between $\overline{\Delta L'}$ and ΔL is not clear yet.

On the basis of the analogy with the hydrodynamics of fluid flow[10], the fluidised beds were classified as follows. The state of stable equilibrium, appearing as uniformly fluidised beds (pulsations rapidly decay), requires certain experimental conditions. Under certain conditions and superficial velocities $w > w_p$ the amplitudes of the turbulent pulsations begin to grow exponentially with time and the disturbances induced hereby propagate in such a way that a chaotic motion of particles begins,

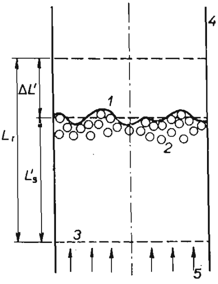

FIG. 1

Sketch of a Nonuniformly Fluidised Bed

1 Top of the bed; *2* particle; *3* grid; *4* wall of the column; *5* inlet of fluid at constant $w > w_p$.

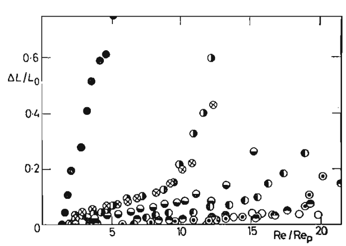

FIG. 2

Experimental Dependence of $\Delta L/L_0$ on Re : : Re_p for Samples Fluidised by Water

Samples: ○ A_3; ⊙ A_4; ◐ A_5; ◑ A_6; ● A_7; ◒ A_8; ◓ A_9; ⊗ A_{10}.

i.e. the small scale instabilities. For w = const., however, the magnitude of the disturbances will grow only up to a certain limit making a spectrum and a bed will be stable on a macro scale. As a consequence, the distribution of the particles will become less uniform, the free configurations and eventually the aggregates and bubbles of fluid will appear. Since the magnitude of the disturbance at w = const. is limited, the other quantities characterizing the bed, such as the voidage, the pressure drop and the height of the bed, cannot take on an arbitrary value either. It may be assumed that the probability of a simultaneous action of all disturbances in the same direction (in a region the cross-section of which is comparable with that of the column) is equal to the probability of their mutual cancelation. Accordingly, the maximum or the minimum height of the bed will be a measure of the sum of all disturbance forces and hereby a measure of induced nonuniformities in the bed. Then the values of such quantities as *e.g.* the height and the voidage will form a spectrum displaying each a typical maximum and minimum. From this concept it follows that an integral measure of all disturbance forces in the bed may be the difference $L_{max} - L_{min} = \Delta L$. Since the disturbances as well as the nonuniformities are distributed in the volume of the bed, it may be assumed that the quantity ΔL is extensive, and, as such, cannot be a measure of the degree of nonuniformity of the bed. As more suitable we take the ratio $\Delta L/L_0$ with the following meaning: The quantity $\Delta L/L_0$ characterizes some average nonuniformity per unit height of the compact bed of particles. The value of $\Delta L/L_0$ increases with growing disturbances and nonuniformities. Two cases may occur: *1.* Under otherwise same conditions the quantity L_0 affects local nonuniformities, *i.e.*

$$\Delta L/L_0 = f(L_0)\,. \tag{2}$$

2. The quantity L_0 does not affect local nonuniformities, *i.e.* ΔL is directly proportional to L_0

$$\Delta L/L_0 \neq f(L_0)\,. \tag{3}$$

We assume that at constant properties of the particles and the fluid, the time variation of the quantities L_{max} and L_{min} represents an ergodic process. In order that the quantity ΔL at constant L_0 may be though of as an integral characteristic of the effect of the disturbances in the bed, the length of the interval of observation, T, must be selected in such manner that the quasi-stationary character of the quantity L'_s can be determined. Examining the stochastic process, $L = f(\tau)$, in inappropriately short time intervals may give markedly smaller value ΔL than that following from the definition. The fluidised beds can be characterized by means of the quantity $\Delta L/L_0$ as follows: If $\Delta L/L_0 = 0$, the layer is uniformly fluidised; if $\Delta L/L_0 > 0$, the layer is nonuniformly fluidised. A comparison of the same particles beds fluidised by liquid and by gas at low pressure is impossible owing to the different mechanism of generation of the fluctuations of the bed height (different structure of the beds).

EXPERIMENTAL

The experimental determination of the values characterizing the fluidised beds was carried out in water–solids system (glass column 0·01148 m in diameter; height 1·01 m; brass grid; screened glass Ballotini and poly(methyl methacrylate) beads ranging between 0·1260 and 2·145 mm in diameter) and air–solids system (perspex glass column 0·1101 m in inner diameter; height 1·55 m; screened glass Ballotini ranging between 0·1265 and 0·4424 mm in dimater). The height of the fluidised beds was determined visually by means of a sliding paper strip. From preliminary experiments with all types of particles and different amounts of solids taken it has been found that the length of the interval of observation, T, which smoothes the quasi-stationarity of the quantity L_s' is approximately 5 minutes. The characteristics of the particles used ared summarized in Table I and the important experimental data in Table II. The experimental values of $\Delta L/L_0$ for water–solids system are plotted as a function of $\mathrm{Re}/\mathrm{Re}_p$ in Fig. 2. A typical dependence of $\Delta L/L_0$ on Re for the air–solids system is shown in Fig. 3.

DISCUSSION

THE CHARACTER OF THE DISTURBANCES

From our experiments it follows that there are two types of disturbances causing the nonuniformity: In liquid-fluidised layers the onset of nonuniformities appears at Reynolds number $\mathrm{Re} \in \langle \mathrm{Re}_{k1}; \mathrm{Re}_{k2} \rangle$, where R_{k1} and Re_{k2} are respectively the first and the second critical Reynolds number[11]. If $\mathrm{Re} < \mathrm{Re}_{k1}$, the character of the flow in the bed is laminar (judging from the effect of the density of fluid on the resistance forces). The turbulence undamped by the bed begins at $\mathrm{Re} = \mathrm{Re}_{k2}$. From this follows that in liquid-fluidised beds the nonuniformities, and hereby the disturbance forces, are caused by the turbulent fluctuations of the velocity of liquid.

TABLE I

Designation and Basic Characteristics of Particles Used

Designation	Particle: Eff. diam.[a] $d \cdot 10^4$, m	Particle: Density ϱ_s, kg/m^3	Designation	Particle: Eff. diam.[a] $d \cdot 10^4$, m	Particle: Density, ϱ_s, kg/m^3
A_1[b]	1·260	2 778·0	A_8[c]	9·040	1 177·0
A_2[b]	2·130	2 726·0	A_9[c]	13·81	1 177·0
A_3[b]	4·650	2 744·0	A_{10}[c]	18·18	1 171·0
A_4[b]	5·190	2 744·3	B_1[b]	1·265	2 615·3
A_5[b]	6·110	2 729·9	B_2[b]	1·528	2 624·9
A_6[b]	11·48	2 737·0	B_3[b]	2·078	2 684·3
A_7[b]	21·45	2 572·0	B_4[b]	4·424	2 697·9

[a] $d = \sum n_i d_i^3 / \sum n_i d_i^2$. Material [b] glass Ballottini; [c] poly(methyl methacrylate) beads.

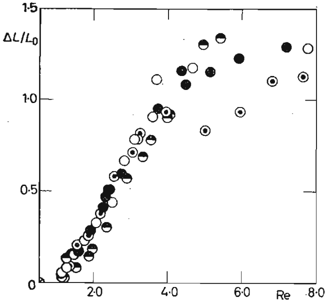

FIG. 3

Experimental Dependence of $\Delta L/L_0$ on Re for Sample B_3 Fluidised by Air at Different Weights of Particles m

⊗ Incipient fluidisation; ○ m 1·50 kg; ⊙ m 2·00 kg; ● m 2·50 kg; ⊖ m 3·00 kg.

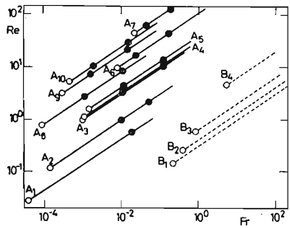

FIG. 4

Plot of Fr *vs* Re for Experimental Systems

○ Incipient fluidisation; ●—● transition region given by $Re_{k1} < Re < Re_{k2}$; ——— water–solids systems; ------ air–solid systems.

TABLE II

Review of Important Experimental Data

Sample	Fluid	Ar	Re_p	Re_{k1}	Re_{k2}	L_0 cm	Maximum value Re	Maximum value $\Delta L/L_0 \cdot 10^2$
A_1	water	30·0	0·027	0·569	nonexistent	3·21	0·976	0·00
A_2	water	167·5	0·130	0·956	2·16	2·41	4·09	0·00
A_3	water	1 613	0·987	3·13	10·0	2·57	19·6	4·28
A_4	water	1 826	1·10	3·51	11·2	4·55	22·2	17·6
A_5	water	2 758	1·59	4·35	13·7	2·73	30·0	25·6
A_6	water	20 790	9·61	16·7	43·7	3·98	121	82·9
A_7	water	134 600	43·7	60·2	123	3·44	327	256
A_8	water	1 280	0·758	2·62	8·36	5·38	16·2	14·9
A_9	water	5 874	3·12	7·12	21·4	3·40	47·6	26·5
A_{10}	water	10 344	5·16	10·3	29·3	6·01	63·7	43·3
B_1	air	196·1	0·151	1·03	2·47	6·03—12·1	4·09	128
B_2	air	348·4	0·252	1·34	3·75	6·01—16·0	5·56	185
B_3	air	906·4	0·591	2·23	7·03	5·87—11·8	7·76	157
B_4	air	8 953	4·54	9·41	27·1	5·84—11·7	16·9	127

In air-fluidised systems at low pressure the nonuniformities, and hereby the disturbances, arise, regardless of the Reynolds number, at velocities just above the incipient fluidisation. On the basis of experimental results it may be concluded that in these cases Re_p is the lower bound of the set of Re numbers at which the layer is nonuniformly fluidised. That means that the disturbances arise outside the bulk of the fluid itself, probably on the boundaries of the system, transferring then into the bulk flow and amplifying there. The chaotic motion of the particles induces the secondary turbulence of the flow of fluid in the bed.

In our experiments ($L_0/D \in \langle 0.55; 1.42 \rangle$) the value of L_0 did not affect the local nonuniformities, *i.e.* ΔL is directly proportional to L_0 and Eq. (*3*) is valid, as indicated by Fig. 3, which is typical for all experimental runs with different amounts of solids, and, consequently, different L_0. By generalizing the data from Table II, Fig. 2 or Fig. 3, one can formulate the following rules for judging the state of the bed on the basis of Re and $\Delta L/L_0$ as a measure of nonuniformity:

In systems fluidised by liquids the bed is uniformly fluidised, *i.e.* $\Delta L/L_0 = 0$, if the Reynolds number satisfies the following inequality

$$Re_p \leqq Re \leqq Re_{k1} .$$

The bed is practically uniformly fluidised, *i.e.* $\Delta L/L_0$ equals 10^{-2} at most, if

$$Re_{k1} < Re \leqq Re_{k2} .$$

The bed is nonuniformly fluidised, *i.e.* $\Delta L/L_0 \gg 0$, if

$$Re > \max(Re_p, Re_{k2}) ,$$

since under the developed turbulence the disturbances are sufficiently great to cause nonuniformities corresponding to $\Delta L/L_0$ of the order 10^{-1} and greater.

In systems fluidised by gases $\Delta L/L_0$ equals zero practically only at $Re = Re_p$, or in close neighbourhood of Re_p. Consequently, the fluidised bed is always nonuniform.

Criteria of Nonuniformity

On adopting the quantity $\Delta L/L_0$ as the criterion of nonuniformity for distinguishing between uniformly and nonuniformly fluidised beds, as well as for judging the degree of nonuniformity (the latter increases with increasing value of $\Delta L/L_0$), then some of the thus far presented criteria can be classified from two different standpoints: Qualitatively, *i.e.* judging on uniformity or nonuniformity of a fluidised system on the basis of the values of the criteria. Quantitatively, *i.e.* judging on the degree of nonuniformity. The experimental values of $\Delta L/L_0$ are compared with other criteria of nonuniformity in Table III.

TABLE III

Review of the Criteria of Nonuniformity in Fluidised Beds

Authors	Designation	Evaluation Eq. or definition
Wilhelm, Kwauk[1]	Fr	w^2/gd
Wicke, Hedden[2]	φ	$w^2\varrho_f/[(\varrho_s - \varrho_f)\, gd\varepsilon^2]$
Tjurjajev, Cajlingold Bujlov[3]	k	$76d^{2\cdot35}[(w/w_p) - 1]^{1\cdot3}$
Harrison, Davidson, de Kock[4]	d_B/d	$\frac{71\cdot3}{\mathrm{Ar}(1-\varepsilon)}\left(\frac{\varrho_s - \varrho_f}{\varrho_f}\right)\left(\frac{\varrho_s}{\varrho_s - \varrho_f} - \varepsilon\right)\left[\left(1 + \frac{\mathrm{Ar}}{54}\right)^{0\cdot5} - 1\right]^2$
Jaskson[5]	$\Delta x/M$	$\eta_1/KM\xi_1$
Rietema[6]	m	$\frac{1}{2(1-\varepsilon)}\left\{1 - 2\varepsilon + \left[1 + 4\,.\,10^3\,\frac{v^2}{d^3g}\frac{(1-\varepsilon)^4}{\varepsilon^6}\right]^{0\cdot5}\right\}$
Molerus[7]	S	$\exp\,[4\cdot1(1-\varepsilon)\, v\lambda/(0\cdot64 + \varepsilon)]/[\varepsilon^3(1-\varepsilon)\,(1-\lambda)\, d^3g)]^{1/2}$

For the lower limit of nonuniformly fluidised beds in water–solids systems we take the transition region delimited by the interval $\mathrm{Re}_{k1} < \mathrm{Re} < \mathrm{Re}_{k2}$; in air–solids systems the region in the proximity of Re_p. We note that the appraisal of individual criteria given in the following cannot be fully generalized for the sake of a limited extent of the experimental data.

Qualitative Appraisal of the Criteria

According to Wilhelm and Kwauk[1], the beds are fluidised uniformly if $\mathrm{Fr} < 1$. As it is seen from Fig. 4, in accord with this all investigated samples of particles in air–solids system should fluidise nonuniformly, which qualitatively agrees with experimental observation. In water–solids systems, however, only some samples should fluidise nonuniformly, and namely at high values of the Reynolds number only (with respect to Re_p), which is not in accord with experimental observation. For instance, at $\mathrm{Fr} = 0\cdot03$ the sample A_7 is still uniformly fluidised, but the sample A_{10} fluidises nonuniformly. Therefore, the Froude number does not contain all quantities characterizing the transition of a uniform bed into a nonuniform one. For all air- and water- fluidised samples of particles the degree of nonuniformity expressed by the quantity $\Delta L/L_0$ increases with increasing value of Fr. It may be concluded that the Froude number qualitatively correctly distinguishes the nonuniformly fluidised water–solids systems from air–solids systems. For the latter the degree of nonuniformity is higher. To give an example, comparing the approximately equal beds of particles fluidised by water (sample A_1) and air (sample B_1) we find that at $\mathrm{Re}_{\mathrm{water}} = \mathrm{Re}_{\mathrm{air}} = 0\cdot9$ the bed A_1 is uniform ($\mathrm{Fr} = 0\cdot49$; $\Delta L/L_0 = 0$) but the bed B_1 is nonuniform ($\mathrm{Fr} = 9\cdot1$; $\Delta L/L_0 = 0\cdot2$).

Wicke and Hedden[2] do not report limiting values for the coefficient φ marking transition from a uniformly fluidised bed into a nonuniform one. The authors only found that the higher the value of φ, the less uniform the bed. This however, contradicts to our experimental finding. At $\varphi = 0{\cdot}1$, for example, all beds should fluidise either uniformly or nonuniformly. But, as seen from Fig. 5, at this value of φ the samples A_1, A_2, A_3, A_7 were still uniformly fluidised, where as the degree of nonuniformity of the samples A_4, A_5, A_6, A_8, A_9, A_{10} increased from the minimum for A_4 ($\Delta L/L_0 = 0{\cdot}011$) up to the maximum value for A_{10} ($\Delta L/L_0 = 0{\cdot}06$). Furthermore, for sample B_1 through B_4 the degree of nonuniformity was very high ($\Delta L/L_0 = 1{\cdot}6$).

As another drawback of the coefficient φ appears that it provides approximately same value of the degree of nonuniformity for a bed fluidised by air and water, which is not in agreement with experiments. For a qualitative classification of the beds the coefficient φ is thus of no practical value. Admittedly, according to Wicke and Hedden, the coefficient φ may retain its significance as a characteristic of the nonuniformity only for spherical particles fluidised by liquids. This opinion, however, may have a practical significance only if certain limiting values for φ are known delimiting transition from a uniform into a nonuniform bed.

Tjurjajev, Cajlingold and Bujlov[3] derived an empirical equation for the calculation of the values of the indicator of nonuniformity, k, by processing an extensive set of experimental data for air–solids systems. Thus it may be speculated that the value of k, or its variations will be in good agreement with our observed degree of nonuniformity $\Delta L/L_0$ in air-solids system. This is confirmed graphically by the log Re $= f(\log k)$ plot in Fig. 6. According to the authors[3], the bed is uniformly fluidised if $w = w_p$ when $k = 0$. At $w > w_p$ we obtain $k > 0$ and the layers become nonuniformly fluidised, which is in agreement with our experiments taking $\Delta L/L_0$ as a measure of nonuniformity. The value of $\Delta L/L_0$ grows initially very slowly and then from certain values $w > w_p$ the growth is faster (Fig. 3). This trend, except in the opposite direction, is followed approximately by the indicator k in Fig. 6. At rapid growth of the indicator k, *i.e.* for small $d(\text{Re})/dk$, the value $\Delta L/L_0$ increases slowly $d(\text{Re})/d(\Delta L/L_0)$ is large, and, on the contrary, at slower

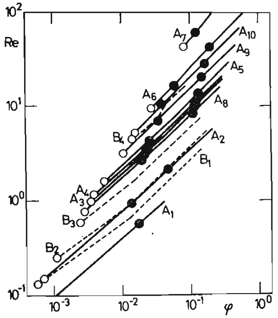

FIG. 5

Plot of φ *vs* Re for Experimental Systems
Same captions as in Fig. 4.

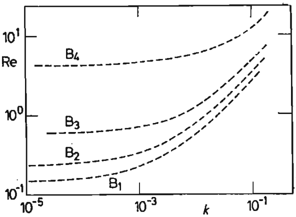

FIG. 6

Plot of the Criterion k *vs* Re for Experimental Air–Solids Systems growth of the indicator k, *i.e.* for greater $d(\mathrm{Re})/\mathrm{d}k$, the value of $\Delta L/L_0$ increases fast ($d(\mathrm{Re})/d(\Delta L/L_0)$ is small). Since the definition equation of the indicator k is empirical and was derived only for air–solids systems, we did not evaluate k for water–fluidised systems.

Harrison, Davidson, and De Kock[4] report as a criterion for nonuniformity the so called stability criterion, d_B/d. The bed is nonuniformly fluidised if $d_B/d > 10$; it is fluidised uniformly if $d_B/d \leqq 1$. For $1 < d_B/d \leqq 10$ the regime of fluidisation is that of the transition from a uniformly fluidised system into a nonuniform one. From Fig. 7 it is seen that this classification is rather rough. Although for water-fluidised systems all samples of particles from our defined transition region (delimited by $\mathrm{Re}_{k1} < \mathrm{Re} < {}_{k2}$) fall into the requested $1 < d_B/d \leqq 10$ region, the measurements at $\mathrm{Re} < \mathrm{Re}_{k1}$, which are far from being nonuniform, should be in the transition region according to d_B/d. For beds fluidised by air the quantity d_B/d takes a different but almost constant value for each sample and accordingly the quantitative evaluation based on d_B/d is impossible. From comparison of the stability criterion with the experiments in Fig. 7, and from considerations which lead the authors[4] to the definition of d_B/d, we regard as more suitable for qualitative characterization of fluidised beds the quantity $(d_B/d)_p$ evaluated from the original definition equation of d_B/d under conditions at incipient fluidisation. Taking ε_p at incipient fluidisation equal 0·42 we can write

$$\left(\frac{d_B}{d}\right)_p = 123\,\frac{\varrho^*}{\varrho_f \mathrm{Ar}}\left[\left(1 + \frac{\mathrm{Ar}}{54}\right)^{0\cdot5} - 1\right]^2, \qquad (4)$$

where $\varrho^* = 0{\cdot}58\varrho_s + 0{\cdot}42\varrho_f$ is the density of the fluidised bed at incipient fluidisation.

A purely qualitative evaluation of the beds is then as follows: If $(d_B/d)_p \leqq 1$, the bed is in a relatively stable hydrodynamic equilibrium in the whole interval $\varepsilon \in \langle \varepsilon_p; 1 \rangle$ and it will appear to be uniformly fluidised. If $(d_B/d)_p > 10$, the bed is in the state of permanently disturbed hydrodynamic equilibrium even just above the incipient fluidisation and it is nonuniformly fluidised. If $1 < (d_B/d)_p \leqq 10$, the bed is in a relatively unstable hydrodynamic equilibrium. This means that from a certain $\mathrm{Re} > \mathrm{Re}_p$, the equilibrium will be disturbed to such extent that the bed will appear to be nonuniformly fluidised.

Let us note that the criterion d_B/d can be a good measure of nonuniformity, however, it is questionable with what accuracy it is expressed by the equation given in Table III. An analogous remark pertains also the criterion due to Jackson and the definition equation discussed in the following.

The values of Jackson's[5] criteria, $\Delta x/M$, were calculated from the equation in Table III, where $K = 2\pi/\lambda = \pi/M$ is the wave number; λ is the wave length of the disturbance; $M = Nd = \lambda/2$ is the half-wave length expressed in terms of particle diameter. In accord with Jackson we took $N = 20$. The values of η_1 and ξ_1 were calculated as the real and the imaginary part of the complex variable S_1 defined by the relation[5]

$$S = (b/2a)\left\{+\,[1 + 4(a - 1)(c/b)^2 - 4\mathrm{i}(c/b)(ae - 1)]^{1/2} - [1 + 2\mathrm{i}(c/b)]\right\}, \qquad (5)$$

$$\text{where } a = 1 + \frac{\varrho_s}{\varrho_f}\left(\frac{\varepsilon}{1 - \varepsilon}\right); \quad b = \left(\frac{\varrho_s - \varrho_f}{\varrho_f}\right)\left(\frac{\varepsilon}{1 - \varepsilon}\right)\frac{\mathbf{g}}{w};$$

$$c = wK/\varepsilon\,; \quad e = 3 - 2\varepsilon\,.$$

It is seen that the calculation of the values of $\Delta x/M$ is rather tedious for common practice A graphical representation of the function $\log \mathrm{Re} = f(\log \Delta x/M)$ is shown in Fig. 8. According to Jackson, if $\Delta x/M = \Delta x/20d > 1$, the disturbances propagate over large distances but the amplification of the amplitude is very slow and the bed will appear uniform. If $\Delta x/M < 1$, the amplitude of the disturbance will amplify several times within a short distance and the bed will appear nonuniform. From Fig. 8 it is seen that from this classification all air-fluidised beds should be nonuniform (which is in accord with experiment), whereas all water-fluidised beds should be uniform (which contradicts to the experiment). Thus it can be concluded that the criterion $\Delta x/M$ is not suitable for judging the uniformity of the bed. As found by Jackson, the maldistribution of the particles in the bed can occur only then if the distance Δx, within which the amplitude of the disturbance increases by a factor of 2·718 (*i.e.* e : 1), is smaller or equal to the height of the bed L_s, at a given porosity of bed, ε (assuming that the disturbance spreads vertically and was originated near the grid). From the above we conclude that it would be more appropriate to classify the beds qualitatively by the quantity $\Delta x/L_r$ (where L_r is the height of the corresponding bed fluidised uniformly at w = const.) rather than by $\Delta x/M$ as suggested by Jackson. By means of $\Delta x/L_r$ the beds can be classified as follows: If $\Delta x/L_r > 1$, the probability of the transition of a uniform bed to a nonuniform one is very small because the distance Δx, within which the amplitude amplifies e-times, is greater than the height L_r. If $\Delta x/L_r \leqq 1$, the bed can become nonuniformly fluidised and the probability of this transition is the greater the smaller the value of $\Delta x/L_r$. A more detailed analysis of the criterion $\Delta x/L_r$ is still under investigation but from preliminary findings this criterion seems to be substantially closer to experiments.

According to Rietema[6], if $m > \varrho_s/\varrho_f$, the system is uniformly fluidised and if $m < \varrho_s/\varrho_f$, the system is nonuniformly fluidised. As seen from Fig. 9 for water-fluidised systems (the ratio $\varrho_s/\varrho_f \approx 2{\cdot}8$, or 1·2), the criterion m is qualitatively only approximate but roughly agrees with the experimental findings. For air-fluidised systems ($\varrho_s/\varrho_f \approx 2{\cdot}65 \,.\, 10^3$), the criterion classifies all systems as nonuniformly fluidised starting from the incipient fluidisation, which again roughly corresponds to the experiments. Since the value of the ratio ϱ_s/ϱ_f in systems fluidised by air is very high (10^3 in the order of magnitude), all beds should, judging from m, fluidise nonuniformly from the incipient fluidisation. For the first qualitative insight whether a certain bed will be uniform or not this criterion is relatively satisfactory.

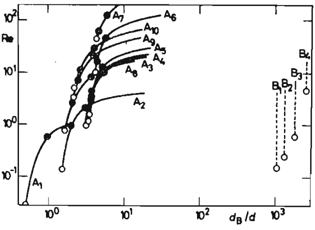

FIG. 7

Plot of the Criterion d_B/d *vs* Re for Experimental Systems

Same captions as in Fig. 4.

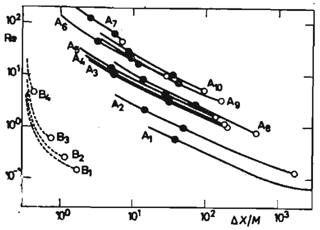

FIG. 8

Plot of the Criterion $\Delta x/M$ *vs* Re for Experimental Systems

Same captions as in Fig. 4.

Molerus[7] has concluded on the basis of a theoretical analysis of the equations of motion of fluidised beds that the state of a uniformly fluidised bed is basically stable (which is in accord with our opinion presented in part Theoretical of this paper). The aptitude for the departure from a uniformly fluidised state was expressed by Molerus by the so called degree of stability, S, (Table III). The smaler the value of S, the greater the ability to upset the stability of the bed resulting in the nonuniform fluidisation. Molerus correctly assumes that there cannot exist a distinct limit expressed by a given degree of stability which would separate the stable and unstable beds or uniformity and nonuniformity fluidised beds. Despite of this the authors sets a limit for distinguishing between the uniformly and nonuniformly fluidising beds equal to $S = 0{\cdot}014$. (The author based this value on experimental observation of the onset of nonuniformities at fluidisation of the lead shot by water ($S = 0{\cdot}012$), and hollow plastic balls fluidised by air ($S = 0{\cdot}015$). For constant value of S its defining equation may be rearranged to give:

$$\lambda = [-(SB)^2 + \{(SB)^4 + [2\nu SB \exp{(A)}]^2\}^{1/2}]/2[\nu \exp{(A)}]^2 , \qquad (6)$$

$$\text{where } \lambda = \varrho_f/\varrho_s;\ A = 4{\cdot}1(1-\varepsilon)/(0{\cdot}64+\varepsilon);\quad B = [gd^3\varepsilon^3/(1-\varepsilon)]^{1/2} .$$

Using Eq. (6) one can calculate λ for different values of ε and to draw the stability diagram of fluidised beds: $\varepsilon = f(\log 1/\lambda)$. This diagram for our systems investigated is shown in Fig. 10. According to Molerus, the uniformly fluidised beds (stable ones) should fall into such interval of bed porosities in which the curve $\varepsilon = f(\log 1/\lambda)$ appears to the right of the $\lambda^{-1} = \varrho_s/\varrho_f = $ = const. line. In the interval of porosities for which the curve $\varepsilon = f(\log 1/\lambda)$ falls to the left of the ϱ_s/ϱ_f = const. line, the beds should be nonuniformly fluidised (unstable beds). By this classification (Fig. 10) almost all samples fluidised by water in the whole interval $\varepsilon \in \langle \varepsilon_p; 1 \rangle$ should fluidise uniformly, which contradicts to the experimental observation. Only for the sample A_7 ($\varrho_s/\varrho_f = 2{\cdot}57$) the bed should be uniformly fluidised in the interval $\varepsilon \in \langle \varepsilon_p; 0{\cdot}69 \rangle$; nonuniformly fluidised in the interval $\varepsilon \in \langle 0{\cdot}69; 0{\cdot}98 \rangle$, and again uniformly fluidised in the interval $\varepsilon \in \langle 0{\cdot}98; 1{\cdot}00 \rangle$. For the sample B_4 fluidised by air the curve $\varepsilon = f(\log 1//\lambda)$ falls to the left

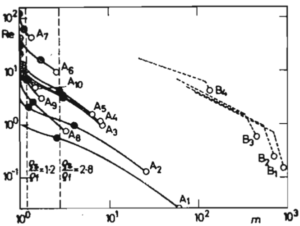

FIG. 9

Plot of the Criterion m *vs* Re for Experimental Systems

Same captions as in Fig. 4.

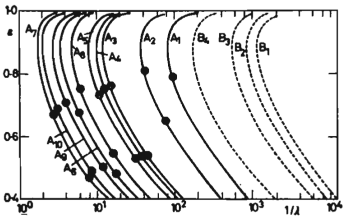

FIG. 10

Stability Diagram by Molerus[7] for Experimental Systems

Between the points ● shown the transition region given by $Re_{k1} < Re < Re_{k2}$.

of the value $\varrho_s/\varrho_f = 2{\cdot}21 . 10^3$ and, consequently, should fluidise nonuniformly in the whole interval $\varepsilon \in \langle \varepsilon_p; 1{\cdot}00 \rangle$. This agrees approximately with the experiments. The samples B_1 through B_3 should be fluidised uniformly from the incipient fluidisation up to a relatively high value of voidage, ε, (particularly for the sample B_1). This, however, does not agree with the experiment at all. The results of comparison of the experimental measurements with the theoretical prediction from the stability diagram thus confirm the correctness of Molerus' assumption that there cannot exist a fixed limit between uniformly and nonuniformly fluidising systems given by a constant value of the degree of stability S.

From the above classification it follows that all criteria considered (except for φ and k) unambiguously distinguish the beds fluidised by air and water, *i.e.* if the value of the criteria increases (Fr, d_B/d, m), or decreases ($\Delta x/M$, S) with the increasing probability of the transition from a uniformly to a nonuniformly fluidised state, then this probability is markedly greater in air–solids systems than in water–solids systems. This rough classification comports also with experimental observation. A finer distinction of the transition of a uniformly fluidised bed into a nonuniform one is

TABLE IV

Values of the Sample Correlation Coefficient, r, between $\Delta L/L_0$ and Individual Criteria of Nonuniformity of Fluidised Beds, and Values of the Student Quantity, t, and the Confidence Intervals of the Correlation Coefficient ϱ

Significance level $\alpha = 0{\cdot}05$; critical value of t-distribution: $t_{n-2,\alpha} = 1{\cdot}96$.

Criterion	r	t	Confidence interval for the correlation coefficient ϱ
	Water-particle, $n = 375$[a]		
Fr	0·73	20·5	$0{\cdot}68 \leqq \varrho \leqq 0{\cdot}77$
φ	0·78	24·0	$0{\cdot}74 \leqq \varrho \leqq 0{\cdot}82$
d_B/d	0·64	16·2	$0{\cdot}58 \leqq \varrho \leqq 0{\cdot}70$
$\Delta x/M$	−0·12	2·4	$-0{\cdot}22 \leqq \varrho \leqq -0{\cdot}02$
m	−0·14	2·6	$-0{\cdot}23 \leqq \varrho \leqq -0{\cdot}04$
S	−0·24	4·8	$-0{\cdot}34 \leqq \varrho \leqq -0{\cdot}15$
	Air-particle, $n = 238$[a]		
Fr	0·80	20·3	$0{\cdot}76 \leqq \varrho \leqq 0{\cdot}83$
φ	0·82	22·2	$0{\cdot}79 \leqq \varrho \leqq 0{\cdot}85$
k	0·86	26·0	$0{\cdot}82 \leqq \varrho \leqq 0{\cdot}89$
d_B/d	−0·15	2·3	$-0{\cdot}25 \leqq \varrho \leqq -0{\cdot}05$
$\Delta x/M$	−0·83	22·5	$-0{\cdot}86 \leqq \varrho \leqq -0{\cdot}79$
m	−0·58	10·8	$-0{\cdot}64 \leqq \varrho \leqq -0{\cdot}50$
S	−0·52	9·3	$-0{\cdot}59 \leqq \varrho \leqq -0{\cdot}44$

[a] n is the total number of processed pairs of values of $\Delta L/L_0$ and a given criteria.

provided by none of the criteria. Only perhaps the criteria d_B/d and m permit approximate determination of the transition in systems fluidised by water. Finally, it can be concluded that if $\Delta L/L_0$ is a correct measure of the degree of nonuniformity, then the other criteria discussed are less suitable and less versatile.

Quantitative Classifiication of the Criteria

In part theoretical it was shown that the quantity $\Delta L/L_0$ can be regarded as a suitable measure of the degree of nonuniformity of fluidised beds. If also the criteria Fr, φ, d_B/d, k, $\Delta x/M$, m, S were to express the degree of nonunifoımity then there should be a close correlation between each of them and the quantity $\Delta L/L_0$. The correlation between individual criteria and the quantity $\Delta L/L_0$ was evaluated on the basis of the sample correlation coefficient, r. The existention of the correlation was tested in each case by the Student test on the significance level $\alpha = 0.05$. For comparison of the correlation of different criteria and the quantity $\Delta L/L_0$ we calculated also the confidence intervals for the correlation coefficient, ϱ. The calculated values of r, t and the confidence intervals for the correlation coefficient of the systems fluidised by water and air are summarized in Table IV. It is seen that the correlation of the investigated criteria with the quantity $\Delta L/L_0$ is greater in general in air-fluidised systems than those fluidised by water. In water-fluidised systems, the closest correlation appears between the quantity $\Delta L/L_0$ and the coefficient of nonuniformity φ, and approximately the same correlation with the Froude number. The second group with a weaker correlation consist of the criteria S and $\Delta x/M$. In air-fluidised systems, the closest correlation appears between $\Delta L/L_0$ and the indicator of nonuniformity, k, which could be expected since its empirical definition equation (Table III) was derived on the basis of experimental measurements and observation in air–solids systems. Into the same group with the indicator k we may put also criteria $\Delta x/M$, φ and Fr. Into the second group with a weaker correlation belong the criteria m and S. The correlation of the quantity $\Delta L/L_0$ with d_B/d is the weakest. If the suitability of individual criteria for qualitative evaluation is not considered then we can conclude on the basis of the sample correlation coefficient that relatively best results in judging the degree of nonuniformity is achieved in systems fluidised by water with the criteria φ and Fr and in systems fluidised by air with the criteria k, $\Delta x/M$, φ and Fr.

LIST OF SYMBOLS

Ar $= [gd^3(\varrho_s - \varrho_f)\,\varrho_f]\mu^{-2}$ Archimedes number
d diameter, or effective diameter of particle, m
d_i diameter of the i-th particle, m
d_B equivalent diameter of bubble in fluidised bed, m
D inner diameter of column, m
Fr $= w^2/gd$, Froude number
g acceleration due to gravity, ms^{-2}

i	imaginary unity
k	indicator of nonuniformity[3]
K	wave number, m^{-1}
L_{max}, L_{min}	maximum and minimum average height of nonuniformly fluidised bed at the time of observation T and at w = const., m
L_0	$= 4m/\pi D^2 \varrho_s$ height of compact bed of particles, m
L_r	height of uniformly fluidised bed, m
L_s	$= (L_{max} + L_{min})/2$ average height of fluidised, m
L_s'	average instantaneous height of nonuniformly fluidised bed, m
$\Delta L'$	$= L_s' - L_r$, m
ΔL	$= L_{max} - L_{min}$, m
m	criterion by Rietema[6]
m	weight of particles in the bed, kg
n_i	number of particles of diameter d_i
M	$= \lambda/2$ half-wave length, m
p	atmospheric pressure, Pa
r	sample correlation coefficient
Re	$= wd\varrho_f/\mu$ Reynolds number
Re_{k1}	$= 0{\cdot}0157\ Ar^{0{\cdot}698} + 0{\cdot}400$ first critical Reynolds number[11]
Re_{k2}	$= 0{\cdot}192\ Ar^{0{\cdot}548} - 1{\cdot}00$ second critical Reynolds number[11]
Re_p	$= w_p d\varrho_f/\mu$ Reynolds number at incipient fluidisation
S	degree of stability by Molerus[7]
S_1	complex quantity defined by Eq. (5), s^{-1}
t	temperature of fluid, °C
T	interval of observation, s
w	superficial velocity of fluid, ms^{-1}
w_p	superficial velocity of fluid at incipient fluidisation, ms^{-1}
Δx	distance within which the amplitude of the disturbance amplifies by a factor of 2·718, m
α	significance level
ε	porosity of bed
ε_p	porosity of bed at incipient fluidisation (minimum-fluidisation voidage)
ξ_1	real part of complex quantity S_1, s^{-1}
η_1	imaginary part of complex quantity S_1, s^{-1}
λ	wave length of the disturbance, m
λ	ratio of density of fluid and particles
μ	dynamic viscosity of fluid, $m^{-1}\ kg\ s^{-1}$
ν	kinematic viscosity of fluid, $m^2\ s^{-1}$
ϱ	theoretical correlation coefficient
ϱ_f	density of fluid, m^{-3} kg
ϱ_s	density of particles, m^{-3} kg
ϱ^*	$= (1 - \varepsilon)\ \varrho_s + \varepsilon\varrho_f$ density of fluidised bed, m^{-3} kg
τ	time, s
φ	criterion of nonuniformity due to Wicke and Hedden[2]

REFERENCES

1. Wilhelm R. H., Kwauk M.: Chem. Eng. Progr. *44*, 201 (1948).
2. Wicke E., Hedden K.: Chem. Ingr. Tech. *24*, 82 (1952).
3. Tjurjajev I. J., Cajlingold A. L., Bujlov A. B.: Chim. Promyšlennost No. *5*, 356 (1961).

4. Harrison D., Davidson J. F., de Kock J. B.: Trans. Inst. Chem. Eng. (London) *39*, 238 (1961).
5. Jackson R.: Trans. Inst. Chem. Eng. (London) *41*, 13 (1963).
6. Rietema K.: Lecture at the International Conference on Fluidisation, Eindhoven 1967.
7. Molerus O.: Chem. Ingr. Tech. *39*, 341 (1967).
8. Aerov M. E., Todes O. M.: *Gidravličeskije i Teplovyje Osnovy Raboty Apparatov so Stacionarnym i Kipjaščim Sloem*. Chimija, Leningrad 1968.
9. Gelperin N. I., Ajnštejn V. G., Kvaša V. B.: *Osnovy Techniki Psevdoožiženija*. Chimija, Moskva 1967.
10. Landau L. D., Lifšic E. M.: *Mechanika Splošnych Sred*. Gostechizdat, Moscow—Leningrad 1953.
11. Beňa J., Ilavský J., Kossaczký E., Neužil L.: This Journal *28*, 293 (1963).
12. Beňa J.: Abstract from the IInd. Conference on Fluidisation Technique, p. 13, Ústí nad Labem 1956.

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